Model for Optimized EVSE Deployment in Dense Urban Areas

Iulia Stamatescu, Roxana Mihalache, Nicoleta Arghira, Ioana Făgărăşan, Grigore Stamatescu Department of Automation and Industrial Informatics University Politehnica of Bucharest, Romania

> . iulia.dumitru@upb.ro

Abstract—Current trends and data show that Electric Vehicle (EV) adoption has significantly increased in the past years in the developed world and will continue to grow towards a 23% market penetration by 2030. An important challenge hindering this mobility transition is range anxiety in direct connection to infrastructure constraints related to the availability, cost and ease of access to suitable charging options. Starting from the existing situation we tackle the issue of new Electrical Vehicle Supply Equipment (EVSE) station installation in urban areas. We present an optimization model that accounts for EV density, usage, battery, capacity and estimated waiting time i.e. user comfort as decision support tool for service providers and city planners. The output yields optimal positions for new EVSE installations by maximising operator profit, user convenience or a weighted balance between the two. Mixed Integer Linear Programming (MILP) is used as a technique to solve the optimization problem with CPLEX/Matlab implementation. The model is suitable for both offline and online running as well as for multi-time period evaluation under dynamic constraint adjustments. The results show the feasibility of our approach based on real data from publicly available repositories such as Plugshare and EV registration records.

Index Terms—electric vehicle, optimization, charging infrastructure

I. INTRODUCTION

Over the last ten years, electric vehicles (EVs) have been gaining market share and increased user acceptance, representing one of the promising solutions for promoting green energy initiatives. According to the US Energy Information Administration (EIA) statistical database, it is possible to highlight the total energy consumption by end users as well as carbon dioxide emissions from developing countries and to state that the transport sector holds about 27% of the energy consumed in the world with a percentage of greenhouse gases of 33.7% [1]. Electric vehicles can become a robust solution for lowering the global emissions of greenhouse gases. Using vehicles powered by electricity, yields not only a cleaner environment, but also reduces user operating costs compared to conventional vehicles. A study conducted on the market of electric vehicles in the USA by [2] indicates that an electric vehicle costs 0.015 USD/1.6 km compared to conventional vehicles at 0.096 USD/1.6 km.

According to the US Department of Energy (USDoE), in the case of conventional vehicles, approximately 15% of total energy consumption is used for starting the vehicle and for powering accessories. Most of the energy consumed is converted to heat during the combustion process which directly and consistently contributes to global warming [3]. On the other hand, electric vehicles consume more than 75% of the energy to run the vehicle. Energy efficiency and the positive impact on the environment are thus the key selling points of electric vehicles.

Nowadays, the number of studies related to EVs is increasing and cover several areas such as: social studies e.g. the influence of sustainable transport on society and the environment, economics e.g. market penetration and economic changes due to the increase of electricity consumption, computer science e.g. computational algorithms for recharging infrastructure management, telecommunications e.g. protocols for communication with charging or payment stations and mobile network infrastructure, electrical engineering e.g. developing less expensive and longer life batteries. Due to the increase in the use of electric vehicles, the effects they have on the environment, the economy and on the national electric system are becoming more significant, including the role of their batteries in balancing local microgrids [4].

Even with the current trend of an evolving electric car market, there are several obstacles that prevent electric vehicles from reaching their full potential, such as: battery storage capacity, battery price, battery life, charging time and availability of electrical vehicle supply equipment (EVSE) [5]. Although the number of public charging stations has increased, they are still not sufficient. Together with the long charging time, this is a major obstacle against widespread adoption.

In this context, this paper presents an optimization model that accounts for EV density, usage, battery, capacity and estimated waiting time i.e. user comfort as decision support tool for service providers and city planners. The output yields optimal positions for new EVSE installations by maximising operator profit, user convenience or a weighted balance between the two. A parallel can be drawn to previous works, for example in information systems to assign virtual machines to servers under capacity constraints [6]. Section II presents the scientific context of the work, alongside recent publications on optimized EVSE deployment. Section III discussed a justification for the chosen problem. Section IV details the formulation of our optimization problem including the objective function and associated constraints. Section V presents the results on a realistic test case within a large city in Europe. Section VI concludes the paper.

II. EVSE DEPLOYMENT - STATE OF THE ART

Since a key element of the adoption of electric cars is a network of high availability charging stations, several studies focus on the development of charging infrastructure. The deployment of electric vehicle charging stations is one challenging issue with the EVSE needing to be placed in the right position. There are two main points factors to consider when planning EVSE placement [7]: grid resources and travel time. From the point of view of a traveler, studies obtain the locations of the charging stations from the traveler's behaviour. These studies can be divided, in turn, into two groups: the development of charging infrastructure between cities and within cities with the general approaches of these groups being different. The development of charging infrastructure between cities generally uses a traffic-based approach [8], while development within cities uses a node-based approach [3]. However, there are exceptions. The reason for these differences is the fact that the limited autonomy of most electric cars currently leads to short trips in the metropolitan area, but currently limits long trips of more than 300km. Therefore, the demand for charging in urban areas occurs at the origin or destination (O-D) of the trip, while, in the case of a trip between cities, it can occur also during the trip. The node-based approach assumes the existence of demand points, which are specific for charging requests within cities. This paper tackles the problem of EVSE optimized deployment within the city of Bucharest, Romania, located at 44.4396 degrees of latitude and 26.0963 degrees off longitude with around 2 Million inhabitants and a metropolitan area of 1811 km^2 .

Other authors, as in [9], provide the data science perspective review of the interdisciplinary area at the intersection of green transportation, energy informatics, and economics. Reference [10] indicates that electric vehicle charging stations should be regarded to comprise multiple types of charging facilities, with different rated charging power, during the planning stage, and a new optimisation model is proposed for the target to minimise the annualised social cost of whole EV charging system. The paper [11] focuses on the return of investments on EV charging stations and proposes a Mixed Integer Linear Programming (MILP) model based on Geographic Information System (GIS) to identify the optimal location of charging stations in cities. Traffic flow data and land-use classifications are used as important inputs, and six important constraints are included in the MILP model with the objective function of maximising the total profits of new charging stations.

The authors of [12] introduce a technique for optimal location and sizing of fast charging stations which minimises the total charging cost taking into consideration transportation loss, grid power loss and build-up costs, Google Maps API, battery state of charge, road traffic density and grid power losses. [13] synthetically considers three indicators of user satisfaction: charging convenience, charging cost and charging time. Considering the load and charging requirements, the model of electric vehicle charging station location and volume is established. Using an artificial immune system algorithm,

an optimised solution of charging station deployment is developed. Reference [14] tackles the charging needs of electric vehicle owners taking into consideration the trade-off between range requirements and battery capacity.

Current studies can be further classified as follows: using real data and using simulated data. A novelty of the paper stems from the fact that optimisation strategies for deployment of EVSE station are rarely using real data which can, with little effort, be tested at large scale in demonstrator sites. There is a growing gap between simulation-based approaches and market facing innovations which can be eventually be integrated into commercially-viable products and services. All the data used in this study is real data which has been retrieved from the PlugShare¹ platform. The algorithm can be replicated with little effort upon any city and can be combined with novel methods for energy forecasting in order to anticipate future demand balancing needs [15].

III. Algorithm description

In order to formulate the optimization problem we first will respond to several questions listed below:

A. For what kind of vehicles is the proposed algorithm suitable?

Cars whose transmission energy is provided only by batteries are known as battery electric vehicles (BEV), as compared to hybrid electric vehicles (HEV) or plug-in hybrid vehicles (PHEV). BEVs can only be operated based on the energy stored in their batteries and their autonomy can vary depending on battery capacity. The battery is also the single largest cost in the manufacturing of the BEV and its capacity becomes a major market differentiator. In general, with a single charge typical BEVs can currently reach between 100km-250km, while the top models can go much longer, between 300km and 500km. This range depends on the pattern of operation and driving style, car configuration, road conditions, weather conditions, battery type and age. Electric propulsion offers high torque and instantaneous response, even at low speeds. These advantages, make them well suited for use in the urban environment, which requires moving at low and medium speeds, with frequent acceleration/deceleration cycles. The present paper considers only cars of BEV type.

B. What type of charging system does the algorithm target?

The EVSE for commercial use is typically composed of three levels of charging power and type. According to SAE EV AC Charging Power Levels, they can be classified as follows.

Alternating Current (AC) Level 1 equipment, often referred simply as Level 1, provides charging at a 120 volts AC plug. Level 1 charging is used usually when is available only a 120V outlet, such as socket at a standard household voltage. Most of the electric cars are fitted in standard with a Level 1 cord. The power used to fully charge an electric vehicle at a Level 1 plug can be approximated with the power used by a toaster.

¹https://www.plugshare.com

Alternating Current (AC) Level 2 equipment, or Level 2, provides charging from a 208V to 240V AC plug. Level 2 charging can be used in both commercial and residential charging. The power used to fully charge an electric vehicle at a Level 2 charging station can be approximated with the power used by a clothes dryer.

DC Fast Charging directly charges the car battery, as compared to AC charging which passes through the EV onboard power electronics systems for conversion, and enables rapid charging along heavy traffic corridors usually used for commercial charging only. The power used to fully charge an electric vehicle at a DC charging point can be approximated with the power used by 5 up to 10 central air conditioners. A comparison of the three supply equipment described above is provided in Table I and Table II.

TABLE I Comparation of Charging Levels - Voltage, Power, Current (adapted from [16])

Charging Level	Voltage [V]	Current [A]	Power [kW]
AC Level 1	120	12 - 16	1.0 - 1.4
AC Level 2	240	< 80	3.6 - 19.2
DC Fast Charging	480	< 125	20 - 72

 TABLE II

 Comparation of Charging Levels - Km per charging hour (adapted from [16])

Charging Level	Power [kW]	Power	Km per	
		similar to	charging hour	
AC Level 1	1.0 - 1.4	Toaster	3.5 to 8	
AC Level 2	3.6 - 19.2	Clothes dryer	16 to 32	
DC Fast	20 - 72	5-10 Central air	80 to 112	
Charging		conditioners		

Another differentiating factor is the plug type: Type 1, Type 2 or one of the main two DC charging standards CCS/ChaDeMo are used. According to Plugshare, in Bucharest there are only 24 charging stations registered on the web site from a total of 148 charging station representing a percent of 16.21%, with a Type 1 plug. There are 91 charging stations registered on the web site from a total of 148 charging station representing a percent of 61,5% with a Type 2 plug. Finally, there are 33 charging stations registered on the web site from a total of 148 charging station representing a percent of 22,29%, for DC fast charging. Analysing Table III which provides a summary of the number of electric vehicle charging station installed in Bucharest by type, it can be stated that the common/used type of supply equipment is AC Type 2. Considering these, the algorithm, used in this paper, proposes optimal positions for new EVSEs of Type 2.

C. Supply equipment cost/site installation

Analysing the public prices offered by the manufacturers of commercial Type 2 charging stations and costs provided in the paper [7] we can conclude that a suitable cost for the deployment of an EVSE currently stands at 2305 Euro. This

 TABLE III

 Number of charging station installed in Bucharest by type

Plug type	Number	Percent
Type 1	24	16,21%
Type 2	91	61,5%
DC Fast Charging	33	22,9%

considers multiple factors such as materials, labor costs for installation, permitting costs and applicable taxes.

IV. FORMULATION OF THE PROBLEM AS A MIXED INTEGER LINEAR PROGRAMMING PROBLEM

This section presents an optimisation algorithm that, taking into account the density of electric cars in a certain region, the charging demand of users in a certain region, user satisfaction and the costs for installation requests provides the optimal positions for the installation of new stations by maximizing the operator profit, the user comfort or a balance of the two. This algorithm can be used as a decision tool for service providers and urban planners and can serve to mitigate local consumption peaks or other types of energy events [17].

The optimization problem was formulated as a mixed integer programming (MILP) problem. The CPLEX utility was used to implement this type of problem. The model is suitable for running both online and offline, as well as for a multiple evaluation with the adjustment of dynamic constraints. The result shows us the feasibility of how we approach the problem, based on real data from publicly accessible directories, such as Plugshare and electric car registration records. The notations used for this section are the following:

- D_i is the demand for EV charging from location i
- c_{ij} location insurance rate i with charging station j i.e. the users satisfaction
- x_{ij} decision to cover the demand of EV charging from location i with a station from the location j
- C_j the capacity of station j (measured in $car \cdot hours/month$)
- A maximum available budget for deployment
- y_j the decision (binary 0 or 1) to build another charging station
- B_j the cost of deployment for the one charging station

The objective function of the proposed optimization problem is described as:

$$\operatorname{Max} f = \sum_{i \in N} \sum_{j \in M} D_i \cdot c_{ij} \cdot x_{ij} \le C_j$$
(1)

Subject to: Constraint 1: The sum of the requests and decisions to satisfy the load of the demand in location j in with a station in location i must be less than or at most equal to the capacity of station j.

$$\sum_{i \in N} D_i \cdot x_{ij} \le C_j \tag{2}$$

Constraint 2: The sum of the costs for requests to install a new station must be less than or at most equal to the maximum possible funding for installation.

$$\sum_{j \in M} B_j \cdot y_j \le A \tag{3}$$

Constraint 3: The amount of decisions to meet the demand for charging electric vehicles at location j with a station at location i must be less than or equal to 1.

$$\sum_{j \in M} x_{ij} \le 1 \tag{4}$$

Defining user satisfaction: A private car in an urban area is usually parked most of the time near the place where the driver carries out his daily activities. Users satisfaction will be measured by access to the charging stations near these locations and weighted by the time spent at each of them.

The concept of immediate coverage is measured every time a given location and reflects the user's demand for charging within an acceptable distance from the nearest public charging station, via the shortest possible route on the network [18]. If this condition is met, then the location is considered to be covered instantly. In this work we will consider that the distance that the user of an electric car is willing to walk to the nearest charging station is similar to the distance that a person considers reasonable a walking distance to a bus station. So, the overall distances between the bus stations are about 400 meters, being generally slightly larger in Europe than in North America. [19] and [20] used in their studies the same radius distance of the coverage area for a bus station.

The degradation of user satisfaction depending on travel distance to/from the EVSE is expressed as:

$$c_{ij} = \begin{cases} 1 & \text{if } d_{ij} \leq d_{full} \\ \frac{d_{max} - d_{ij}}{d_{max} - d_{full}} & \text{if } d_{full} \leq d_{ij} \leq d_{max}, \ \forall i \in N, j \in K \\ 0 & \text{if } d_{ij} \geq d_{full} \end{cases}$$
(5)

where

- *N* represents the set of zones
- K is the set of possible charging station locations
- c_{ij} the rate of securing the location *i* with the charging station *j*
- d_{ij} the distance from location *i* to the charging station *j*

To represent the empirical behaviour, a three-segment function in Figure 1 is considered to evaluate the user satisfaction degradation based on distance:

- first segment with an user satisfaction of 1 (100%) for distances below the full satisfaction distance (d_{full})
- the second segment with a linear satisfaction degradation rate, from 1 to 0, between the full satisfaction distance and the maximum (partial) satisfaction distance (d_{max})
- and without any satisfaction for distances greater than the maximum allowable distance

Other user satisfaction curves can be evaluated such as piece-wise exponential functions and s-shaped curves depending on personalised user preferences.



Fig. 1. User satisfaction

V. RESULTS

The algorithm was implemented in the CPLEX environment. CPLEX is a high-performance mathematical programming solution for linear programming, mixed integral programming and quadratic programming. CPLEX Optimizer provides the power to solve large scale real-world optimization problems with good multi-platform computational performance. CPLEX offers numerous algorithms to solve linear programming problems e.g. the primary or dual simplex algorithm, the barrier algorithm and the network algorithm can be chosen. The barrier (or interior point) algorithm provides an efficient approach especially in the case of large problems with rare values. CPLEX can very effectively solve arrays with rare values. A preprocessor is used to reduce the size of the problem before solving it, sometimes by an order of magnitude. CPLEX is very robust and efficient.

The CPLEX branch-and-bound algorithm for solving mixed integral programming problems uses modern functions such as the cutting plan and heuristic method to find complete solutions. In combination with the state-of-the-art preprocessor, these features make CPLEX a very powerful tool for solving large and difficult problems of mixed full programming. CPLEX provides a Benders decomposition algorithm, which can be used to solve linear problems with a decomposition structure, including stochastic programming problems with integer variables in the first phase.

A. Case Study 1 - Validation of the algorithm on a minimum test network

We present a test case study by applying the optimization model on a small scale charging network, which can be scaled up to cover the whole city. For testing the algorithm the network includes the nodes S1, S2, S3, S4, S5 equivalent to the charging point placement. The algorithm was validated using the following input data:

Nodes	Charging demand	Spatial coordinates x-y	Station capacity	
	[vehicles/day]	[km]	[vehicles/day]	
S1	20	0.21 - 0.24	15	
S2	4	0.21 - 0.1	3	
S3	2	0.42 - 0.99	3	
S4	2	0.66 - 0.83	3	
S5	1	0.59 - 0.6	5	

 TABLE IV

 INPUT DATA FOR THE OPTIMIZATION ALGORITHM

TABLE V INPUT CONSTANTS

Constants	Values	
Cost per station	2305 Euros	
Maximum budget	23000 Euros	

A simplifying assumption is that charging demand is expressed in the number of charges per month, without considering the average time per charge. The test example can be mapped onto a 2×2 cell subset from the real data, while considering that the cells are squares with 500 m sides. The network is analyzed by calculating the distance between any two nodes, the node with the highest demand is selected and an optimal output is provided to lessen the burden on it by choosing nodes where the demand–distance metric is optimal. Figure 2 graphically depicts the outcome of the algorithm on the test case. In this case, S1 is first chosen and the nodes



Fig. 2. Test network diagram

that are within an acceptable distance are selected. That leads to the demand in node S1 to be transferred to node S2 at a distance of 0.14 km and to node S5 at a distance of 0.42km. The distance to nodes S1 and S4 are considerably higher at 0.77km and 0.74 km respectively. Next step concerns the node S2 with the second highest charging load, which is redirected to nodes S3 and S4, accounting for the demand constraints on nodes S1 and S5.

B. Case Study 2 - Validation of the algorithm on the Bucharest network

Based on real collected data, the charging demand for the city of Bucharest is analyzed. Figure 3 illustrates the charging demand distribution. The starting point of the algorithm was



Fig. 3. Map with the Demand for the City of Bucharest

the available charging station already deployed in Bucharest in Figures 4 and 5. The map has been divided into equally sized, square, cells covering the metropolitan area of the city.



Fig. 4. Bucharest Map with the Charging Station

After completing the data collection and structuring, the validation of the algorithm on the Bucharest city network taking into account all the restrictions specified above was carried out. Initially, the result were unsatisfactory due to the fact that for the existing infrastructure currently the user satisfaction restriction that involves a maximum travel of 0.4 km it cannot be satisfied because the distance between the stations is much greater. Taking into account these results obtained initially, we gradually relaxed the restrictions in order to be able to offer a feasible solution for the current case study. Restrictions were relaxed by the modification of the user satisfaction function, thus: d_{full} was adjusted to 0.8 and d_{max} was adjusted to 1.6.

In this case, following the application of the algorithm, 49 modifications from the initial network were obtained.



Fig. 5. Map with the Distribution of Charging Station in the City of Bucharest

The results were organized in table VI under the following structure, where:

- *i*, *j* represent the ID-s of the states associated with the request
- the distance between 2 stations (road <i-j>) was calculated based on the coordinates of each station
- the decision represents the suggestion to increase or not (1-yes, 0-no) the loading capacity of the station from location i

TABLE VI Algorithm output for Bucharest case study

St. i	St. i	St. j	St. j	Road x-y	Connection	Dec.
	demand		demand		i to j	
St. ID	v/day	St. ID	v/day	km	-	
45	1	90	25	1.01	<45 90>	1
44	1	39	6	0.61	<44 39>	1
34	1	112	25	1.09	<34 112>	1
32	1	19	500	0.57	<32 19>	1
31	1	7	15	0.29	<31 7>	1
30	1	37	375	1.01	<30 37>	1
28	4	84	25	1.03	<28 84>	1
26	1	86	10	1.17	<26 86>	1
44	5	42	115	1.21	<24 42>	1

VI. CONCLUSIONS

We presented an optimization model for EVSE installation in dense urban areas. The model accounts for EV density in a given area, user demand for the charging service, user satisfaction and installation costs for the network operator. The output consists of the optimal positions for installing new EVSE stations by maximizing operator profit, user comfort or a weighted sum of the two. The model has been tested on real data from a public infrastructure platform and can be useful as a decision tool for utility providers and city planners. Future work concerns a scalable approach for automated parametrisation and evaluation, with programmatic access to heterogeneous data sources.

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