

Dynamic Nonlinear Modelling of Building Structure Using the Force Analogy Method

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Abstract — The nonlinear analysis of civil engineering structures is still a topic of high significance for structural engineers. The use of an explicit time integration method is investigated in this paper. The time integration scheme is derived from the state-space model of the equation of motion. The Force Analogy Method (FAM) is used to account for physical nonlinearity of a 10-story reinforced concrete (RC) frame structure. Two earthquake recorded ground motions are used in this paper in the attempt to catch the accuracy of the investigated numerical scheme. The results are compared and validated with well established commercial software. The model described in this paper will ease and simplify the overall process of testing the structural behavior to a wide range of recorded and artificial ground motions.

Keywords — *explicit analysis; nonlinear analysis; plastic rotation; reinforced concrete (RC) frame; Simulink*

I. INTRODUCTION

The Force Analogy Method (FAM) has been developed in recent years as a fast tool to estimate the nonlinear behavior of civil engineering structures [1], [2]. The Force Analogy Method (FAM) focuses on a change in displacement, not in stiffness, to give the same level of force. Consequently, the initial stiffness matrix of the structure is used throughout the analysis. Using the small displacement assumption and the principle of superposition, the total displacement is decomposed into two independent components, namely an elastic and a plastic component. Giving these, the physical nonlinearity is treated as a perturbation of the excitation forces in the equation of motion.

The continued research of Wong et al. [3]–[6], Li G. et al. [7]–[9], Bahar H. [10], Iraj T. [11], Safaei S. et al. [12] and Ji-Ting Qu [13] in the past twenty years conducted to the refinement of the method and to its application in considering the physical and geometric nonlinearity for frames and shear elements, stiffness and strength degradation, for both steel and reinforced concrete structures. Another civil engineering application in which FAM has proved to be efficient is the analysis of the pounding phenomena [14].

The first aim of this study is to check the possibility of using a variable time-step integration procedure. All the current studies using the FAM use the state transition matrix to calculate the nonlinear dynamic structural response. The procedure is indeed numerical efficient since the state transition matrix is calculated once for a constant time step. However, the use of a constant time-step may become time-consuming if the certain degree of accuracy is required. The second aim of this study is to introduce the use of Simulink in civil engineering applications. The use of Simulink may prove to be efficient, especially when the control theory applications are investigated [15]. By now, there are very few papers in civil engineering applications in which Simulink capabilities are used [16]–[18]. Finally, the third aim of this study is to check the FAM procedure accuracy by comparing the dynamic response in terms of story displacements with commercial software data. The paper is addressed mainly to civil engineering community and to those scientists performing in the structural control field.

A 10 story and 2 bay RC frame is used in this paper. The plane model is acted by two earthquake recordings namely 1986 Mexico City NS component and 1977 Vrancea NS component. The corresponding dynamic response is obtained considering the full model of the structure.

II. STATIC MODELLING USING FORCE ANALOGY METHOD

In this section is presented the theoretical background of building modelling using FAM [1]. In FAM the resisting force term ($K(u) * u = K\tilde{u}$), is regarded as the difference between the resisting forces corresponding to a total elastic response $u(t)$ and the forces producing a plastic displacement $u''(t)$. Thus, the displacement vector is split in two components, according to (2), while the initial stiffness is kept constant:

$$K\tilde{u} = p \quad (1)$$

$$\tilde{u}(t) = u(t) - u''(t) \quad (2)$$

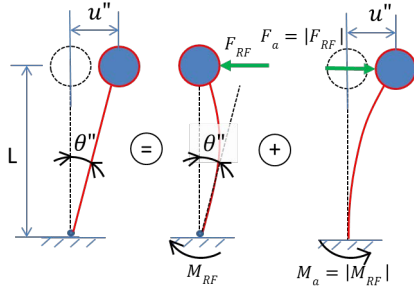


Figure 1. Force Analogy Method for SDOF (adapted from [1])

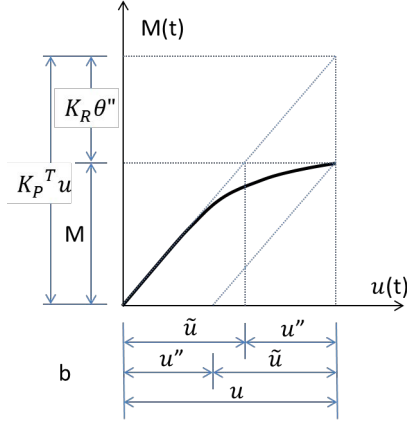


Figure 2. Moment at different level of structural displacement (adapted from [2])

Fig. 1 and 2 illustrate the concept of the FAM for a single degree of freedom (SDOF), and introduces the member force recovery matrix K_P and member restoring force matrix K_R [2]. The plastic rotation at the base of the cantilever beam is denoted by θ'' . To represent the permanent deformation u'' as a force one will imagine that a force is applied to restore the structural displacement back to its original position.

$$F_{RF} = Ku'' = -\frac{3EI}{L^3}u'' \xrightarrow{u''=L\theta''} K_P = \frac{3EI}{L^3} \quad (3)$$

$$M_{RF} = F_{RF}L = -\frac{3EI}{L}\theta'' \leftrightarrow K_R = \frac{3EI}{L^3} \quad (4)$$

In a multi-degree of freedom structure (MDOF), the member force recovery matrix K_P and member restoring force matrix K_R for a framed structure can be composed by assembling the corresponding matrices in local system, i.e. $K_{P_{el}}^{local}$ and $K_{R_{el}}^{local}$. Because the axial force in the columns influences the capacity of the plastic hinges located on columns, an member axial force matrix $K_{N_{el}}^{local}$ is defined for each element. These matrices can be obtained from the elastic stiffness matrix K_{el}^{local} , as shown in (5):

$$K_{el}^{local} = \begin{bmatrix} \frac{ES}{L} & 0 & 0 & -\frac{ES}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{ES}{L} & 0 & 0 & \frac{ES}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \end{bmatrix}$$

$$K_{P_{el}}^{local} = \begin{bmatrix} 0 & 0 \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ \frac{4EI}{L} & \frac{2EI}{L} \\ 0 & 0 \\ -\frac{6EI}{L^2} & -\frac{6EI}{L^2} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \quad K_{N_{el}}^{local} = \begin{bmatrix} -\frac{EA}{L} \\ 0 \\ 0 \\ \frac{EA}{L} \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

$$K_{R_{el}}^{local} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$$

III. DYNAMIC MODELLING OF THE BUILDING ACTED BY AN EARTHQUAKE RECORDING

The equation of motion of a physical nonlinear structural system is given in (6). Giving the change in the displacement vector, the equation of motion (6) can be rewritten in (9). The state space formulation is employed to obtain a set of 1st order differential equations (10),

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\tilde{\mathbf{u}} = \mathbf{F} \quad (6)$$

where $\tilde{\mathbf{u}}(\mathbf{t}) = \mathbf{u}(\mathbf{t}) - \mathbf{u}''(\mathbf{t})$ and $\mathbf{F}(\mathbf{t}) = -\mathbf{M}\mathbf{i}\mathbf{a}_g(\mathbf{t})$, while \mathbf{i} is the influence matrix of the ground acceleration vector $\mathbf{a}_g(\mathbf{t})$.

The mass matrix \mathbf{M} of the structure is composed by assembling the corresponding element mass matrix \mathbf{M}_{el}^{local} in (8). \mathbf{M} is also corrected by terms corresponding to the distributed load on beams. The S parameter represent the cross-section area, the L parameter represents the length of the element. The parameter γ is the mass of the unit volume, namely $\gamma = 2.5 \text{ tons/m}^3$. I is the moment of inertia of the cross-section. Furthermore, the damping matrix \mathbf{C} of the structure is obtained by assuming a proportional (Rayleigh) type damping.

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \quad (7)$$

$$M_{el}^{local} = \gamma \frac{SL}{2} \begin{bmatrix} \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{26}{35} & \frac{11L}{105} & 0 & \frac{9}{35} & -\frac{13L}{210} \\ 0 & \frac{11L}{105} & \frac{2L^2}{105} & 0 & \frac{13L}{210} & -\frac{L^2}{70} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{9}{35} & \frac{13L}{210} & 0 & \frac{26}{35} & -\frac{11L}{105} \\ 0 & -\frac{13L}{210} & -\frac{L^2}{70} & 0 & -\frac{11L}{105} & \frac{2L^2}{105} \end{bmatrix} \quad (8)$$

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{i}\mathbf{a}_g(\mathbf{t}) + \mathbf{K}\mathbf{u}''(\mathbf{t}) \quad (9)$$

$$\begin{bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{i} \end{bmatrix} a + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{K} \end{bmatrix} \mathbf{u}''$$

where $\dot{\mathbf{z}} = \begin{bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$,
 $\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{i} \end{bmatrix}$, $\mathbf{F}_p^c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{K} \end{bmatrix}$

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}a + \mathbf{F}_p^c \mathbf{u}'' \quad (10)$$

The procedure of obtaining the inelastic displacement \mathbf{u}''_{k+1} is quite straightforward. First, a trial bending moment is calculated, considering the state vector \mathbf{z}_{k+1} and no plastic rotation increment exist. The trial bending moment values obtained are one by one compared to their capacities. If at a certain plastic hinge the bending moment capacity is exceeded, the value is cut to the limit according to the $\mathbf{m} - \boldsymbol{\theta}''$ relation. A plastic rotation increment will thus need to be calculated for that position. A reduced matrix $\mathbf{K}_{R1}^{\text{reduced}}$ is calculated in each step in order to invert and to obtain the plastic rotations. This matrix is obtained by extracting those lines and columns for which plastic rotation increment is expected to be calculated. After the plastic rotation vector is obtained, the bending moment redistribution due to the plastic rotation is performed. Prior to the calculation of the plastic displacement \mathbf{u}''_{k+1} , the bending moments obtained after the redistribution are again compared to the corresponding capacities. A second iteration may be required in limited cases. In the end of the procedure the inelastic displacement \mathbf{u}''_{k+1} is calculated, and its values are used to calculate the state vector at $k+2$ step.

Please note that using (10) and the procedure¹ illustrated in Fig. 3, the quantities defining the equilibrium at a certain time step are calculated in a two-step procedure: first obtain the state vector \mathbf{z}_{k+1} , and secondly the plastic displacement \mathbf{u}''_{k+1} is calculated based on the state vector and several step iterations illustrated in Fig. 3, which also depend of the trial bending moment used. For this reason, the previous step equilibrium is used in this paper, namely the trial bending moment is calculated using (13).

$$\begin{aligned} \mathbf{m}_{k+1} &= \mathbf{K}_p^T \mathbf{u}_{k+1} - \mathbf{K}_R \boldsymbol{\theta}_{k+1}'' \xrightarrow{\Delta \boldsymbol{\theta}''=0} \\ \mathbf{m}_{k+1}^{\text{trial}} &= \mathbf{K}_p^T \mathbf{u}_{k+1} - \mathbf{K}_R \boldsymbol{\theta}_k'' \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbf{m}_{k+1}^{\text{trial}} &= \mathbf{K}_p^T (\mathbf{u}_k + \Delta \mathbf{u}) - \mathbf{K}_R \boldsymbol{\theta}_k'' \Rightarrow \\ \mathbf{m}_{k+1}^{\text{trial}} &= (\mathbf{K}_p^T \mathbf{u}_k - \mathbf{K}_R \boldsymbol{\theta}_k'') + \mathbf{K}_p^T \Delta \mathbf{u} \end{aligned} \quad (12)$$

$$\mathbf{m}_{k+1}^{\text{trial}} = \mathbf{m}_k + \mathbf{K}_p^T \Delta \mathbf{u} \quad (13)$$

¹The notations used in Fig 3 are specific to Matlab/Simulink and Octave platform.

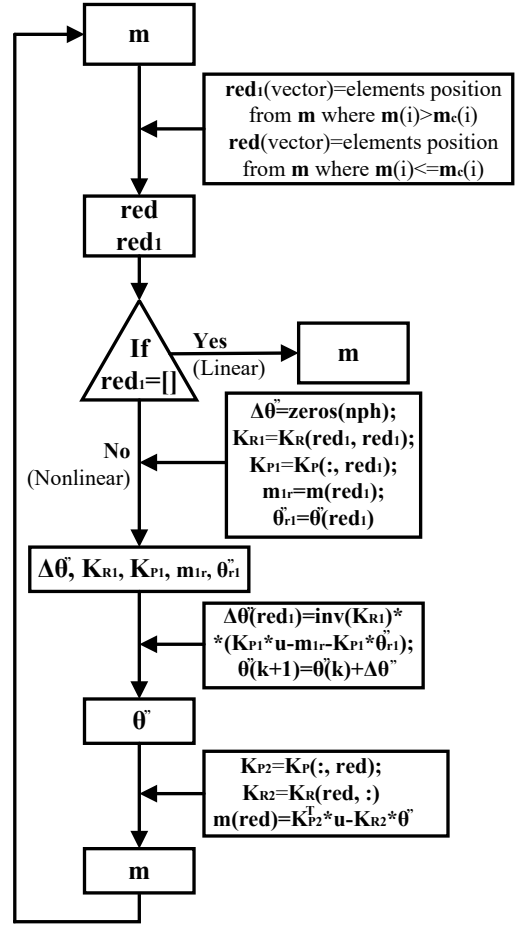


Figure 3. Nonlinear function (see block (e) from Fig. 5)

IV. CASE STUDY - 10 STORY BUILDING

A. System Description

The system consists of a 10 story, 2 bay RC frame structure. The details of the system are illustrated in Fig. 4. The material considered is reinforced concrete having the Young's modulus $E = 3 \cdot 10^7 \text{ kN/m}^2$ and $\gamma = 2500 \text{ kg/m}^3$. An uniform distributed gravitational load $p = 20 \text{ kN/m}$ is considered acting on all beams. The cross-section of the beams is considered $60 \times 30 \text{ cm}$ and the cross-section of the columns is considered $80 \times 100 \text{ cm}$. The story height is considered $H = 4 \text{ m}$ and the total width of the structure is $2L = 12 \text{ m}$. The coefficients α and β in (7) are 0.3734 and respectively 0.0047, which correspond to considering a 5% damping ratio on the first two eigenmodes.

The plane system consists of 33 joints, each of having three degrees of freedom. The total number of degrees of freedom (DOF) is 99, out of which 9 are zero because of the restraints (in joint 1, 12, 23 the vertical and horizontal displacement and the rotation is restrained). In each time sequence, in each of the rest of 30 joints, the horizontal displacement, the vertical vertical and the

rotation are calculated. Also, the state vector includes another 90 components related to the above mentioned velocities. The state matrix A has the dimension of 180×180 , and the state vector z has the dimension of 180. The plastic displacement vector include 90 components only, corresponding to the first 90 components of the state vector.

Each of the 50 elements depicted in the sketch of the numerical model (Fig. 4) has two ends. The internal resultants (axial force and bending moment) are calculated in each of these 100 ends of elements ($nph = 100$). Thus, in each time-sequence 100 axial forces and 100 bending moments are calculated. The representation in Fig. 8, 9 and denoted by #61 represents the cross-section where the bending moment bending moment at the left end of the element 31, which is located above the ground floor level.

Because of paper length limits, only the top displacement of the structure is illustrated in Fig. 8 and 9. This displacement is positioned in the state vector on position 28, 58 or 88. The horizontal displacement of the joints located at the same level is the same because of the big axial stiffness of the beams.

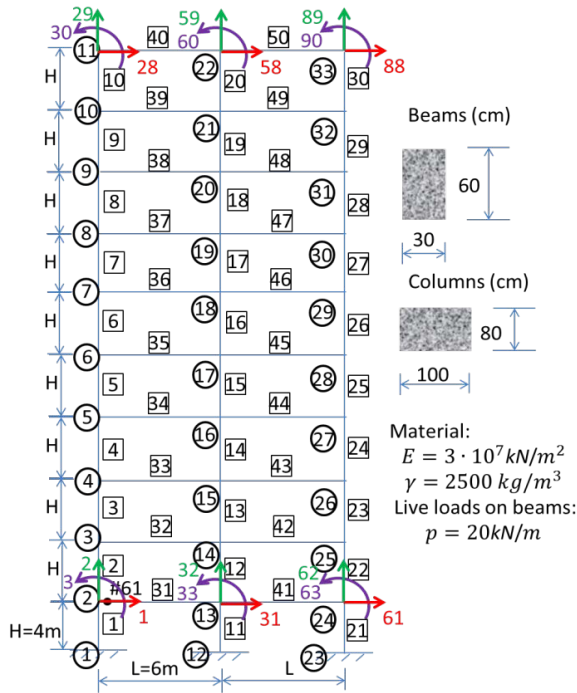


Figure 4. Sketch of the numerical model

B. System Model

The Simulink diagram (Fig.5) consist of several blocks and functions interconnected, as follows:

(a) - consists of the earthquake recording, measured in terms of ground acceleration

(b) - is the input matrix B used to distribute the acceleration to the masses located at each joint (second term in (10))

(c) - represents the first term in (10) - Az

(d) - represents the third term in (10) - F_p^c

(e) - represents the nonlinear function illustrated in Fig. 3

(f) - is the function that calculates the axial force in elements and corrects the bending moment capacity vector when an interaction curve is defined. In this case study the bending moment capacity vector is considered constant throughout the dynamic simulation.

(g) - the integrator block

The numerical integration method used is "trapezoidal rule and backward differentiation formula" with variable-step. The step-size time-history is illustrated in Fig. 6 and 7. The relative and absolute tolerance used is 0,1. The step number is 7576 for Vrancea recording and 13353 for Mexico City recording.

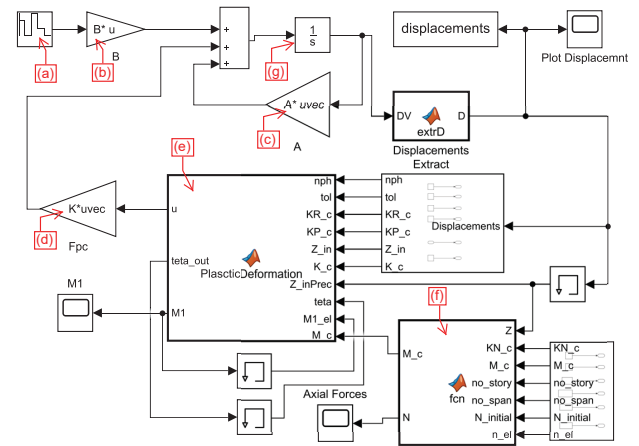


Figure 5. Simulink diagram for dynamic model

C. Simulation Results

The system described in this case study is acted by two earthquake recordings, namely 1986 Mexico City NS component and 1977 Vrancea NS component. The corresponding dynamic response obtained with Simulink is compared with the results of SAP2000 commercial software. The ground motion acceleration timestep is 0.005s. The time duration for the two analyses is 37,5s for Vrancea NS and 66,4s for Mexico City NS respectively.

Fig. 8 and 9 illustrate the earthquake recording, top story total and plastic displacement time-history and bending moment in plastic hinge #61 time-history. One may observe a good match between the case study results and the reference ones.

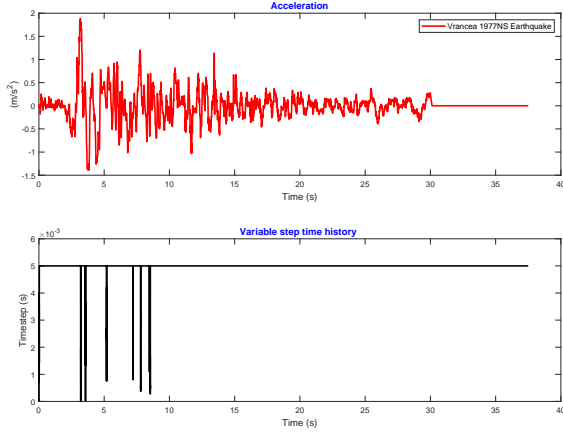


Figure 6. 1977 Vrancea NS - step-size time history

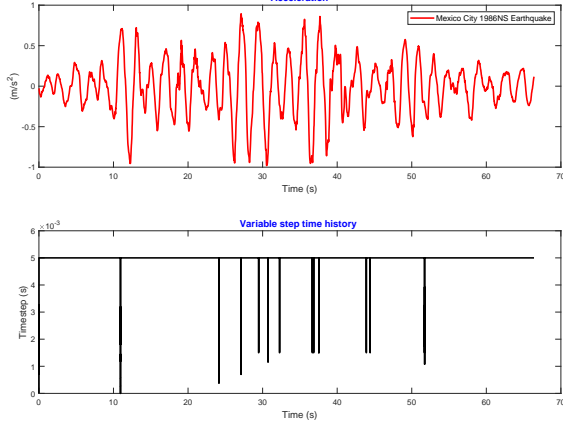


Figure 7. 1986 Mexico City - step-size time history

The mean absolute percentage error (MAPE) is used to analyze the simulation results accuracy. The MAPE method is described in 14.

$$MAPE = \frac{100\%}{N} \sum_{i=1}^N \left| \frac{y_i - \hat{y}_i}{y_i} \right| \quad (14)$$

where

- y_i - the reference value
- \hat{y}_i - the simulation value
- N - the number of values

The MAPE method is applied to the simulation results obtained using both "1977 Vrancea NS" and "1986 Mexico City" ground motion recordings. The simulations error obtained are expressed in percentages and shown in Table I. One may observe that the calculated error is very small, both for displacement and axial force. The bending moment error increases as the displacement error increases. Still, a less than 5% error is reasonable, taking into account that the reference is not obtained by instrumentation of the real behavior of the structure, but using another numerical method.

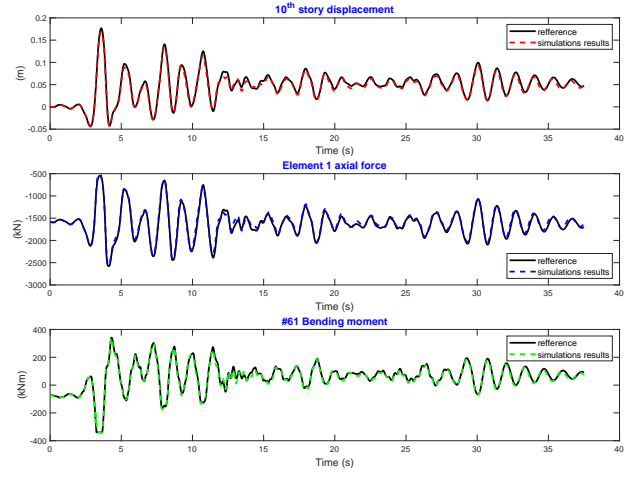


Figure 8. 1977 Vrancea NS simulation results

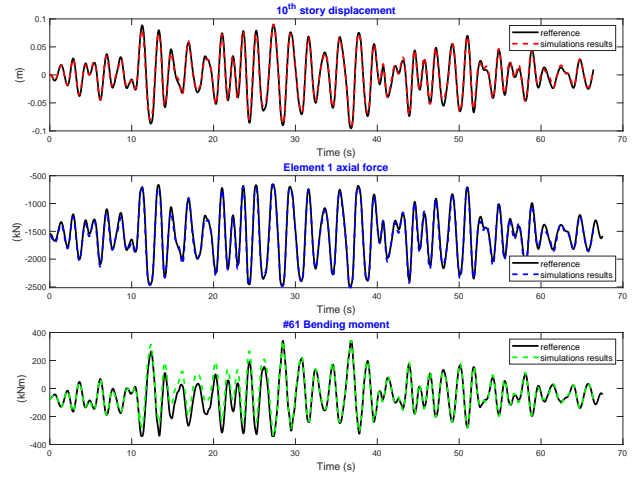


Figure 9. 1986 Mexico City simulation results

Table I. PRECISION ERROR (MAPE - METHOD)

Earthquakes	10 th story displacement	Element 1 axial force	#61 Bending moment
1977 Vrancea NS	0.3 [%]	0.02 [%]	1.67 [%]
1986 Mexico City	0.95 [%]	0.03 [%]	4.83 [%]

V. CONCLUSION

The smart buildings erected in the 21st century should be properly modeled. Because the structures are designed from economical reasons, some nonlinear behavior of these is expected. Consequently, their behavior in earthquake events should be completely determined. This is accomplished currently by commercial software in which the second order differential equation of motion of the structure is directly integrated using implicit integration schemes techniques.

The earthquake recording is an unknown input in civil engineering applications and for this reason many

simulations are required to capture the overall dynamic behavior of the structure. Numerical efficiency is thus a fundamental request for civil engineering purposes.

In this paper the force analogy method is employed to model the nonlinear behavior of a reinforced concrete frame structure. The case study structure is typical for a building located in seismic area cities. The numerical integration scheme is explicit and, in conjunction with force analogy method, conducts to high numerical efficiency.

In the current version of FAM, the researchers use explicit procedures involving state transition matrix to calculate the structural response. The use of state transition matrix for nonlinear problems is a cumbersome procedure since high nonlinearities or numerical instabilities may arise in certain time-steps. It should be worth to mention that in civil engineering applications, the numerical conditioning number, calculated as the ratio of 1^{st} to last singular value is in the magnitude of $10^6..10^8$.

The numerical data obtained is compared and validated with reference data obtained with well-established commercial software.

The program detailed in this paper can be easily extended to account for a wide range of structural layout. Moreover, the program detailed can extended to account for different ground motion, either recorded or artificial.

An overall picture of the dynamic behavior of cities acted by earthquakes can thus be obtained through statistical means.

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